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An evaluation framework for a project-based learning mathematical modeling task on mortgage repayment: linking arithmetic and geometric sequences with present value reasoning

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Abstract. This paper presents a Project-Based Learning (PBL) mathematical modeling task for upper-secondary students and outlines an accompanying evaluation framework suitable for research paper submission. The task uses a mortgage repayment decision to connect arithmetic sequences (equal-principal repayment) with geometric series via present value discounting (fixed-payment/annuity repayment), and requires students to justify a recommendation under an affordability constraint (payment-to-income ratio). We specify research questions, implementation procedures, assessment instruments (a short conceptual test, an artifact-based rubric, and an optional perception survey), and a mixed-method analysis plan (paired comparisons, rubric scoring, and qualitative coding of typical modeling difficulties). Empirical findings from classroom implementation are intended to be reported in a subsequent paper following deployment of this framework.

Keywords: project-based learning, mathematical modeling, study design, sequences, spreadsheet visualization

1. Introduction

Mathematical modeling is widely recognized as a key competency for connecting school mathematics with real-world decision-making [1]. Curriculum standards call for students to engage in modeling cycles: understanding context, making assumptions, mathematizing, solving, interpreting, and validating [2]; international perspectives on modelling in mathematics education align with this view [3]. In upper-secondary curricula, arithmetic and geometric sequences bridge algebraic structure and quantitative reasoning [4], with finance applications offering rich modeling tasks [5]. Project-Based Learning (PBL) deepens engagement by integrating content with real-world problems [6], and the integration of mathematics with real-world contexts is also emphasized in STEM and modelling perspectives [7]. Yet assessment of modeling competence remains challenging, requiring both product- and process-based evidence [8]; recent work also examines structured modelling tasks in authentic contexts [9]. Meanwhile, financial literacy has gained international attention [10,

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11], with measurement and policy frameworks [12] and evidence on its economic importance [13]; but few studies link sequences explicitly to financial modeling in a research-ready framework.

This paper presents a reusable PBL modeling task on mortgage repayment, connecting arithmetic sequences (equal-principal scheme) with geometric series (annuity scheme via present value discounting). Specifically, we investigate: Equation (1) how the task supports students to distinguishing arithmetic and geometric structures and interpreting present value; Equation (2) what levels of modeling quality are evidenced in student artifacts; and Equation (3) what typical difficulties arise and which instructional supports help resolve them. The study employs a mixed-method design combining pre/post conceptual tests, artifact-based rubric scoring, and qualitative coding of modeling difficulties. By providing a transparent and reusable evaluation framework—including instruments, procedures, and analysis plans—this work enables future classroom implementation and replication. Empirical results will be reported in a subsequent paper following deployment.

2. Conceptual framework, task design, and evaluation plan

2.1. Conceptual background, design rationale, and research questions

We adopt a modeling view emphasizing the modeling cycle: understanding the context, simplifying and assuming, mathematizing, solving, interpreting, validating, and communicating. Evidence of modeling competence can be collected from both process (reasoning, justification, revisions) and product (reports, spreadsheets, charts, decision memos). Mortgage repayment is a suitable context because it offers structure recognition (linear decrease vs geometric discounting), a conceptual hurdle ("averaging money over time" vs present value reasoning), and a decision under constraint (affordability vs total interest). Figure 1 summarizes the modeling cycle adopted for the task.

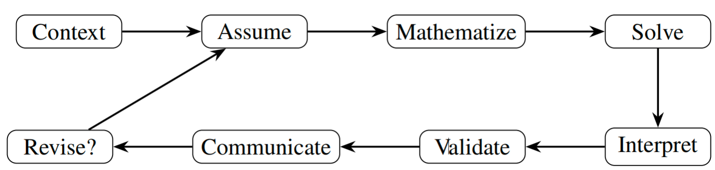

Figure 1. Modeling cycle (simplified) adopted for the task

Figure 1 illustrates the simplified modeling cycle used in this task. The evaluation framework is guided by three research questions. RQ1 (Conceptual understanding): How does the task support students to distinguish arithmetic and geometric structures and to interpret present-value discounting in the annuity model? RQ2 (Modeling quality): What levels of modeling performance are evidenced in student artifacts (assumptions, derivations, computation, interpretation, and decision justification)? RQ3 (Difficulties and supports): What typical modeling difficulties arise (e.g., the misconception that one can average future payments without discounting), and what instructional supports help address them?

2.2. Task design and intervention

Students receive a mortgage scenario with principal $P = 2.7 \times 10^6$ (yuan), term $n = 300$ months (25 years), monthly interest rate $r = 0.003$, monthly household income 28,000 yuan, and an affordability guideline that monthly payment over income not exceed 50%. Two repayment schemes are compared: equal-principal repayment (constant principal each month plus interest on the remaining balance) and fixed-payment (annuity) repayment (constant monthly payment derived from present value). Figure 2 illustrates the resulting payment patterns over time (equal-principal: linearly decreasing; annuity: constant); it was generated in Python using the formulas presented in Section 4 (12-month mini-example: $P = 12,000$, $n = 12$, $r = 0.005$).

Each group submits a modeling report (assumptions, variables, derivations, results, decision memo), a spreadsheet (computations and charts), and a short presentation or one-page executive summary. The design assumes a constant interest rate r, payments at the end of each month, and no early repayment, fees, or delinquency.

2.3. Mathematical model (reference solution outline)

2.3.1. Equal-principal repayment (arithmetic structure)

Monthly principal (Equation (1)):

$$M = \frac{P}{n} \tag{1}$$

Remaining balance before month k (Equation (2)):

$$B_k = P - (k-1)\frac{P}{n} = P\left(\frac{n-k+1}{n}\right) \tag{2}$$

Interest in month k (Equation (3)):

$$I_k = rB_k = \frac{Pr(n-k+1)}{n} \tag{3}$$

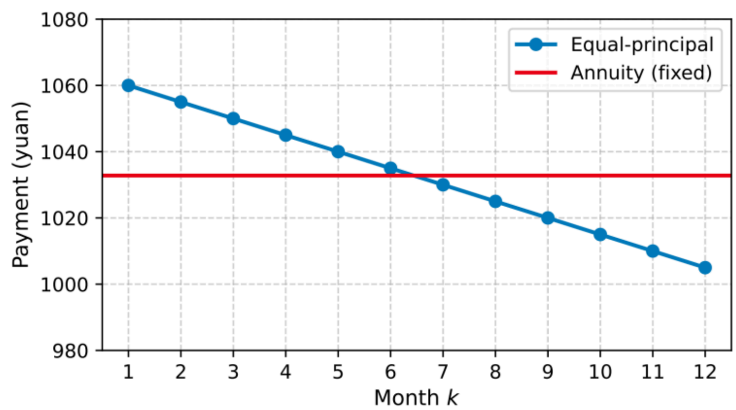

Figure 2. Payment pattern: equal-principal (decreasing) vs annuity (constant). Exact values from formulas; 12-month mini-example

Figure 2 shows the comparison of monthly repayments for the two patterns.

The total payment is shown in Equation (4):

$$x_k = M + I_k = \frac{P}{n} + \frac{Pr(n-k+1)}{n} \tag{4}$$

Difference is shown in Equation (5):

$$x_{k+1} - x_k = -\frac{Pr}{n} \tag{5}$$

hence $\{x_k\}$ is an arithmetic sequence (linearly decreasing).

The total interest is shown in Equation (6):

$$I_{EP} = \sum_{k=1}^{n} I_k = \frac{Pr(n+1)}{2} \tag{6}$$

2.3.2. Fixed-payment (annuity) repayment (geometric structure via Present Value (PV))

Present value of a future payment F due in t months is shown in Equation (7):

$$PV(F,t)=\frac{F}{(1+r)^t} \tag{7}$$

Equivalence principle is shown in Equation (8):

$$P=\sum_{t=1}^{n}\frac{x}{(1+r)^t} \tag{8}$$

Solving yields:

$$x=\frac{Pr(1+r)^n}{(1+r)^n-1},\qquad I_{FP}=nx-P \tag{9}$$

2.3.3. Worked numerical example (transparency check)

Substituting $P=2.7\times10^6$, $r=0.003$, $n=300$ into Equation (6) and Equation (9):

$$I_{EP}=\frac{Pr(n+1)}{2}=1,219,050\ yuan,$$
$$x\approx 13,662.07\ yuan/month,$$
$$I_{FP}\approx 1,398,621.97\ yuan.$$

Affordability is shown in Equation (10):

$$x_1^{EP}=\frac{P}{n}+Pr=17,100\Rightarrow 61.1\%\ (>50\%),\qquad x^{FP}/28,000\approx 48.8\%\ (\le 50\%) \tag{10}$$

2.3.4. Extension and enrichment

To deepen the mathematical content and provide extensions for advanced students, the following extensions can be integrated into the task or used as optional enrichment. Each sub-section below develops the mathematical details and suggests natural questions for group work or homework.

(1) Geometric series: full derivation.

The annuity equivalence (8) is the present value of n equal payments x; the sum is a finite geometric series with first term $a=x/(1+r)$ and common ratio $q=1/(1+r)$ (with $q\neq 1$ since $r>0$). We used the Equation (11)

$$\sum_{t=1}^{n}q^t=q\,\frac{1-q^n}{1-q} \tag{11}$$

the result obtained by using $q=(1+r)^{-1}$ is shown in Equation (12)

$$\sum_{t=1}^{n}\frac{1}{(1+r)^t}=\frac{1}{1+r}\,\frac{1-(1+r)^{-n}}{1-(1+r)^{-1}}=\frac{1-(1+r)^{-n}}{r} \tag{12}$$

Hence $P=x\cdot\frac{1-(1+r)^{-n}}{r}$, so the final answer is shown in Equation (13)

$$x=\frac{Pr}{1-(1+r)^{-n}}=\frac{Pr(1+r)^n}{(1+r)^n-1} \tag{13}$$

This derivation makes the geometric-series structure explicit: the annuity formula is exactly the application of the sum of a finite geometric sequence. Students can be asked to derive the sum $\sum_{t=1}^{n}q^t$ from $S_n=q+q^2+\cdots+q^n$, $qS_n=q^2+\cdots+q^{n+1}$, and $(1-q)S_n=q-q^{n+1}$, then substitute $q=(1+r)^{-1}$ to obtain the present-value factor.

(2) Recursive (difference-equation) view and closed form.

For the annuity, the remaining balance B_k immediately after the k -th payment (with $B_0=P$) satisfies (Equation (14))

$$B_{k+1}=(1+r)B_k-x,\quad k=0,1,\ldots,n-1 \tag{14}$$

So $\{B_k\}$ is a first-order linear recurrence. The homogeneous part has general solution $B_k^{(h)} = C(1+r)^k$; a particular solution constant in k gives $B^{(p)} = x/r$. So $B_k = C(1+r)^k + x/r$. From $B_0 = P$ we get $C = P - x/r$, hence (Equation (15))

$$B_k = P(1+r)^k - \frac{x}{r}\left((1+r)^k - 1\right) \tag{15}$$

Setting $B_n = 0$ yields $P(1+r)^n = (x/r)((1+r)^n - 1)$, i.e. $x = Pr(1+r)^n/((1+r)^n - 1)$, consistent with (9). Thus the same annuity formula can be obtained either from the sum of discounted payments or from solving the recurrence; the recurrence (14) also gives the amortization table row by row via Equation (15) ($B_0, B_1, \ldots, B_n$ and the corresponding interest $I_k = rB_{k-1}$, principal $x - I_k$). This dual perspective (closed form vs. recursive) connects to discrete dynamical systems and reinforces algebraic consistency.

(3) Sensitivity and comparison in detail.

Total interest comparison. From (6) and (9), we have $I_{EP} = Pr(n+1)/2$ and $I_{FP} = nx - P$. Their difference is (Equation (16))

$$\Delta I = I_{FP} - I_{EP} = nx - P - \frac{Pr(n+1)}{2} \tag{16}$$

Substituting (9) into Equation (16) shows that $\Delta I > 0$ for $r > 0$ and $n \geq 2$ (annuity always costs more in total interest than equal-principal for the same P, r, n). Students can verify this numerically and, if calculus is included in the curriculum, check that $\partial \Delta I/\partial r > 0$ and $\partial \Delta I/\partial n > 0$ for typical parameters.

Sensitivity of monthly payment to rate. The annuity payment (9) is increasing in r : a higher interest rate implies a higher monthly payment for the same principal and term. Students can explore via a spreadsheet how much x changes when r increases by 0.1% or 0.5%, or plot $x(r)$ for fixed P and n. Similarly, for a fixed P and r, x decreases as n increases (longer term, smaller monthly payment, but more total interest).

Decision under constraint. Under the affordability rule $x \leq 0.5 \times income$, the maximum affordable principal with annuity is $P_{max} = (0.5 \times income) \bullet \frac{1-(1+r)^{-n}}{r}$. So the feasible set of (P, n, r) is constrained by this inequality; students can discuss how a change in income or rate shifts the feasible region.

(4) Early repayment: modeling and interest saved.

If the borrower makes an extra one-time payment Q at the end of month k, the remaining balance just after that month becomes $B'_k = B_k - Q$ (where B_k is the balance that would have applied without the extra payment). Two natural options are: (i) keep the same monthly payment x and pay off the new principal B'_k over fewer periods; or (ii) keep the same remaining term $n-k$ and reduce the monthly payment to x' satisfying $B'_k = x' \frac{1-(1+r)^{-(n-k)}}{r}$, i.e. $x' = B'_k r(1+r)^{n-k}/((1+r)^{n-k} - 1)$.

Interest saved. The remaining interest without early repayment is $I_{remain} = (n-k)x - B_k$ (total future payments minus remaining principal). With extra payment Q, the new remaining interest depends on the choice (i) or (ii). For option (ii), the new remaining interest is $I'_{remain} = (n-k)x' - B'_k$; the interest saved is $I_{remain} - I'_{remain}$. Students can implement both options in a spreadsheet and compare total interest saved under different k and Q, linking to "what if" analysis and financial literacy.

(5) Optional: Perpetuity and growing annuity (enrichment).

If the number of payments is infinite and each payment is x , the present value becomes a perpetuity: $P = x/r$ (the sum $\sum_{t\geq 1} x/(1+r)^t = x/r$ when $|(1+r)^{-1}| < 1$). This is the limit of the annuity formula as $n \to \infty$: $\frac{1-(1+r)^{-n}}{r} \to 1/r$. As a further extension, a growing annuity with payment $x_t = x_0(1+g)^{t-1}$ at time$t$and discount rate r (with $r \neq g$) has present value $P = x_0 \frac{1-((1+g)/(1+r))^n}{r-g}$. These formulas connect the mortgage task to infinite series and more advanced financial topics, making them suitable for high achieving students.

2.4. Study design and evaluation

2.4.1. Participants and setting

(1) Target grade and rationale

The task is designed for Grade 11 or 12 (upper-secondary; students typically aged 16–18). At this level, arithmetic and geometric sequences and series are usually already introduced, and the mortgage context supports both consolidation of these topics and a first encounter with present-value reasoning. The task can be adapted for Grade 10 if sequences have been covered earlier in the curriculum.

(2) Proposed implementation (concrete design)

A feasible, research-ready design is:

·Participants: one or two intact classes, approximately 40–55 students total (e.g., 2 classes of 20–28 each). Convenience sampling within the school is assumed; demographics and prior exposure to sequences/finance can be reported in the implementation paper.

·Grouping: 3–4 students per group, yielding about 12–18 groups. Small groups facilitate collaborative spreadsheet work and discussion, while keeping the total number of artifacts manageable for rubric scoring.

·Duration: 4 sessions of 45–50 minutes each, plus a short pre-test (about 10 min) before Session 1 and a post-test and artifact collection (about 10 min) after Session 4. Total in-class time is roughly 200–220 minutes (e.g., four double-periods or four single periods over two weeks). The implementation process of the course is shown in Table 1.

Sites may adjust class size, number of sessions, or session length; the instruments and analysis plan remain applicable as long as the core task (two schemes, affordability constraint, report + spreadsheet) is preserved.

2.4.2. Procedure

(1) Classroom format: whole-class and group work

Each session follows a launch–explore–share structure: students first receive a minimal common basis (scenario, definitions, one worked step), then work in groups on derivations, spreadsheet tasks, or decision memos, and finally share and compare their work. This structure is common in inquiry-based and Problem-Based Learning (PBL) designs (e.g., the NCTM-style launch-explore-summarize framework). Figure 3 summarizes the cycle; Figure 4 outlines the full four-session flow with Pre and Post (Table 1).

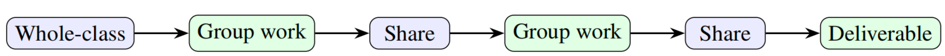

Figure 3. Classroom format: one full flow from whole-class input through two rounds of group work and share to deliverable. Light blue: teacher-led; light green: small-group work

Table 1. Implementation timeline (four sessions). Each session follows a launch–explore–share structure to support inquiry and group discussion

Phase	Activities
Pre (~10 min)	Conceptual pre-test; optional baseline survey.
Session1 (45–50 min)	Launch: scenario and assumptions; whole-class introduction to equal-principal repayment. Explore: groups derive x_k (4), verify arithmetic structure (5). Share: brief report-back and comparison.
Session2 (45–50 min)	Launch: spreadsheet setup for equal-principal; introduction to PV and annuity. Explore: groups work on annuity formula (9) and equivalence (8). Share: compare with equal-principal (total interest, payment pattern).

Table 1. Continued

Session3 (45–50 min)	Launch: affordability constraint (payment/income $\leq 50\%$). Explore: group decision task (which scheme? justify); spreadsheet charts (payment over time, interest share). Share: peer feedback and teacher clarification.
Session4 (45–50 min)	Launch: memo structure and rubric. Explore: groups write decision memo and one-page summary; finalize report and spreadsheet. Share: optional short presentation; collect artifacts.
Post ($\sim$ 10 min)	Conceptual post-test; perception survey; collect group artifacts.

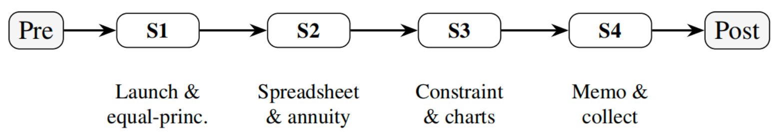

Figure 4. Full course flow: Pre → Session 1 → Session 2 → Session 3 → Session 4 → Post. S1–S4: launch–explore–share in each session (see Table 1)

(2) Refined discussion cycle: discuss–report–discuss again

Within each group-work phase, a finer cycle can be used: groups first discuss (derive, compute, or interpret); then one or more groups report briefly to the class; the teacher or peers give feedback, after which groups discuss again to revise or deepen. Figure 5 spells out this cycle so that "group work" in Figures 3 and 4 can be read as containing one or more such rounds.

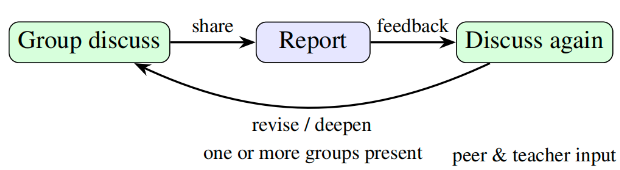

Figure 5. Refined cycle within a group-work phase: discuss → report (to class) → discuss again (revise or deepen). One or more rounds can be run before moving to the next whole-class input

2.4.3. Instruments

I1: Short conceptual test (pre/post). Targets: structure identification/justification, PV discounting interpretation, and constraint-based decision reasoning. An example item blueprint: (1) Prove $\{x_k\}$ is arithmetic for equal-principal repayment using constant difference. (2) Explain why "monthly payment = total/n" is invalid for fixed-payment repayment; use PV reasoning. (3) Under an income constraint (payment/income $\leq 50\%$), justify a recommendation using numbers and reasoning.

I2: Artifact-based rubric. Group report + spreadsheet rated on: assumptions/variables, mathematical correctness, tool use/visualization, interpretation/decision, communication.

I3: Optional perception survey. Brief Likert items on modeling confidence and usefulness of spreadsheet visualization.

2.4.4. Analysis plan

Quantitative: paired t -test (or Wilcoxon) for pre/post; report effect size (Cohen's d). Rubric: descriptive statistics; optional correlation with gains. Rater agreement: percentage agreement at minimum; optionally Cohen's κ or ICC.

Qualitative: code artifacts (and brief observation notes if feasible) for typical difficulties and revisions. A preliminary coding scheme includes: Averaging Money Across Time (AVG-MONEY)—treating payments across time as directly additive/average (ignoring PV); Present Value Direction Reversed (PV-DIR)—reversing PV direction (multiplying by $(1+r)^t$ instead of dividing); Sequence Justification Missing (SEQ-JUST)—claiming arithmetic/geometric without difference/ratio justification; Graph Description Without Model Link (GRAPH-DESC)—describing graphs without linking to model structure; Constraint Use in Decision (CONSTR-USE)—explicitly using affordability constraint in decision argument.

2.4.5. Note on reporting

When the framework is implemented, results will be reported with clear separation between (i) feasibility and process observations and (ii) outcome measures. Until then, this manuscript should be read as an evaluation-ready design report.

3. Conclusion

We present a reusable PBL mathematical modeling task on mortgage repayment and provide a transparent evaluation framework suitable for research dissemination. The paper specifies instruments and analysis plans to support future implementation and replication, while deliberately avoiding outcome claims until the framework is deployed. We hope that other researchers and instructors can adapt both the task and the evaluation design to their own contexts. However, this manuscript does not report classroom outcome data; it documents an evaluation-ready framework. The minimal design may involve one class and convenience sampling, limiting generalizability. Short tests may have limited reliability, and rubric scoring requires rater calibration. Without a control group, causal interpretation would remain cautious even after deployment.

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